1 Reversibility of exact calculation by saturation and an incommensurability budget for coprime rings in 7 bits

Subtitle: Coordination through dis-coordination in icosagonal polar topologies.

Epigraph — *"Truth is told with topology, not with adjectives."*

Author (Principal Investigator): Rodolfo Leiva **Affiliation:** Sustrato AI Research Lab (Independent Researcher) **ORCID:** 0009-0003-4251-2733

DOI: 10.5281/zenodo.22099183 · https://doi.org/10.5281/zenodo.22099183

How to cite this work: Leiva, R. (2026). *Reversibility of exact calculation by saturation and an incommensurability budget for coprime rings in 7 bits.* Sustrato AI Research Lab. Zenodo. https://doi.org/10.5281/zenodo.22099183

Master version (v0.6) — first delivery of the sustrato.ai collective. Text under **Creative Commons Attribution 4.0 (CC BY 4.0)**. The code cited in Appendix A is distributed under **Apache License 2.0**: the text license does not cover the scripts. This document is verifiable: every result cites its script (Appendix A). Whoever wants to check runs the code; whoever wants to argue has something concrete to argue with.

1.1 Abstract

We present a system of exact, reversible calculation —the Fractal Saturation Grammar (FSG)— that operates with integer arithmetic, without floating point and without positional zeros. The engine processes numbers of arbitrary length with a bounded register (7 bits per cell) and no drift: in a ten-million step reversibility test, engine drift is zero, against 3.61×10 for the 64-bit float (measured up to 10 M steps; beyond that, extrapolation). A theorem bounds the simultaneous return of coprime rings: **T 20 · M(N)**; in the N=99 case, five one-byte cells determine a cycle of **82,645,608,260 exact beats** — literal incommensurability produced by integer mathematics, verified without simulating. The same framework re-expresses the round of the Maya calendars (759,200 days) as a case of the theorem, without attributing our vocabulary to their record. The human-AI collaboration that produced these results is described in Methods.

1.2 1. The test at the gate

Before the theory, a measured datum.

Five nested pentagons rotate on the polar map of twenty rays, each with an incommensurable frequency: [3, 5, 7, 11, 13]. The system adds one step at a time; then it is reversed and we subtract the same steps. The question is falsifiable: *does the engine return exactly to the starting point?*

Steps	Engine drift (FSG)	64-bit float drift
10,000,000	**0**	3.61×10

The float rounds by design; the engine does not round by construction — it keeps what is left over instead of discarding it (§3). The difference is not a matter of degrees of precision; it is a matter of the architecture of the exact. The script that produces this table runs in seconds (Appendix A; `e8_reversibility.py`).

This inversion of the opening is deliberate: the conceptual thesis that follows explains why this difference matters, but the proof is already on the table from the first page.

1.3 2. The category error

Modern numerical practice suffers a category error: **confusing the resolution of the instrument with the resolution of the phenomenon.** It manifests on two axes.

The axis of space — the decimal as a non-existent distinction. If two states fit in the same cell of the instrument, they are the same state at the resolution of the phenomenon. Adding decimal digits below the cell size adds no real resolution: it introduces an external tiebreaker the system does not possess. On a wide-cell logistic map, a thousand points fall in the same cell; iterated one step, the real physics has a single destination — but with twenty-eight decimal digits, the interpolation produces **a thousand ghost trajectories**: parallel stories that do not exist at the resolution of the phenomenon. We call this distinction a trap — the one the instrument introduces where the phenomenon does not have it (the decimal trap), and its temporal twin (the clock trap).

The axis of time — the clock as verdict. The sampling interval Δt is not a property of the phenomenon; it is a property of the instrument. The heart does not beat in twenty frames; magnetic resonance measures in twenty frames. Two field cases, one lesson, with verifiable arithmetic:

- *Cerebral aneurysm (flow MRI).* Domain of two millimeters; blood at forty

centimeters per second; one frame every 45 ms. Between frames, blood travels 18 mm — nine times the domain. The Courant number is nine: any trajectory reconstructed by interpolation passes through points where the blood never was, and coherent structures are computed upon that fiction.

- *Loop current, Gulf of Mexico.* Frames every three hours; the particle

travels eleven kilometers between frames. When Courant exceeds one, joining two frames is inventing the path.

In both, the lesson is the same: when Courant exceeds one, the structures measured on the interpolated trajectory are those the literature calls **Lagrangian coherent structures** (Haller 2015) — the robust phenomenon, measured over fiction.

The cross-demonstration. Same phenomenon, two instruments: one takes eight frames per turn, the other forty. The computed velocity changes almost fivefold (4.9×; hand-verifiable arithmetic on the same turn) depending on who looks — that belongs to the apparatus. The number of turns is identical in both — that belongs to the phenomenon. What survives the change of instrument is a **reparameterization invariant** —a topological invariant—; what changes belongs to the realization layer. The community knows these pieces —Courant, shadowing, LCS—, yet operates the Courant number as a control knob of a simulation. Faced with instrument data, Δt is not a knob: it is a verdict. And the verdict is kept, without drawing the conclusion.

The corrective principle, in one line: assert only invariants. Everything that depends on the clock of the instrument, or on digits below the cell size, belongs to the realization layer — not to the model of the phenomenon.

Nature already seems to choose this filter: the systems that persist in biology and in the skies avoid resonances of small cycles, and coprimality —the condition the engine audits— is the signature of that persistence (we will see it in a shell in §5 and in the calendars in §7).

1.4 3. The grammar

The Fractal Saturation Grammar (FSG) is a finite set of local rules that generates numbers of arbitrary length by recursion: **every cell that saturates adds one to the next level.** The rule is universal and without exceptions; cells process each level with the same constant cost (O(1) per level).

The count. Each cell holds a symbol from 1 to 99, without the zero digit and without multiples of ten: exactly 90 valid symbols. The closure labels at 99; the count is of 90 positions (it could be another count in another instance — the logic is not of those numbers). Zero does not enter the cell: nothingness is not a position. Nine hundred ninety-nine is not written with a zero; it is written with levels.

What saturates, rises. Nine plus one, in units, does not produce a "10": the cell fills, saturates, and a cell is born to the left. It is the car odometer brought to arithmetic: the units roller reaches its top and its own motion turns the tens roller. The number grows to the left, adding whole levels — never to the right, adding invisible portions. Magnitude is tuple depth, not decimal width.

The bounded register. The consequence is stark: the working register never grows. A pair of cells per operation, whether the number has one cell or sixty-two. Each cell fits in seven bits; an eight-bit microcontroller processes numbers of arbitrary length in streaming, without ever loading the complete number (§8). The operations are equally frugal: addition, subtraction and the residue cell work with peak 181 (eight bits); multiplication and division, with peak 8278 (sixteen bits) — always bounded, regardless of the length of the number.

The § cell — the surplus is kept. When a division does not close, the excess is not discarded nor turned into infinite decimals: it is kept in a separate cell we write §, with cap equal to the divisor. The invariant is exact:

real $\times$ cap + residue = exact value. Always. Not a single unit is lost.

At rest, § never holds more than cap$-$1; its transient peak is $2 \cdot$ cap$-$2. A byte is more than enough. That cell —the residue made first-class passenger— is the same figure that will reappear as biological debt in §5 and as calendared surplus days in §7. What is not enough to saturate is kept exact, not discarded.

The frontier — the bridge to the world. Inside the engine there are no decimals, no floats, no injected irrationals. The decimal lives only at the frontier, in what we call the **customs**: the point of communication with the world, an open frontier that translates — and that stamps the passage: crossing costs no computation, it costs declaring. Whoever crosses presents their resolution, as one presents a passport; the stamp is the integrity control, not a toll. The customs rule is non-negotiable: **the question must declare with what resolution it wants to see the result.** If the context asks to divide 10 by 3 in decimal, the customs says so, it does not disguise it. The output leaves in integer pairs — numerator and denominator, two columns, not a floating point. Whoever receives decides with what resolution to collapse the fraction, and that decision is recorded on their side, not hidden from ours. The customs does not forbid the decimal: it forces it to declare itself.

The passage is cheap and bounded. The translation operates in time **O(D)**, with D the tuple depth: linear in the length of the number, not exponential. And the number crosses as what it is — a sequence of cells, a text of symbols, not the materialized integer. The output Horner (base 90, cell accumulator, carry of at most one) keeps the register per operation within seven bits even for the unrepresentable numbers of §6: verified up to the grain of rice k25, 10 ^{2} in twenty-two cells, without the integer ever existing in full (experiment E17, §4). The giant integer is not generated to then check whether it fits: it is not generated; it is only verified, and verification is part of the integrity control, not of the flow.

Supports are finite on both sides. Granted "seven bits inside", the question about the output is natural. The answer is the same symmetry: there is no infinite computer memory, no infinite Eulerian mesh, no infinite pixels. The decimal produces twenty digits and the support collapses it to one pixel. The continuum has exactly the same problem — it just does not declare it. Our customs collapses once, at the end, with a declared bound (at the pixel render, half a cell; verified at three canvas resolutions, §8, experiment E20); conventional rendering collapses at every chained operation, without a record. The same residue, named in one case and unnamed in the other (§8).

Coprimality as a filter. Two integers are coprime if they share no prime factors. On twenty rays, an integer frequency has period 20/gcd(frequency,20): coprime with twenty visits all twenty rays; degenerate, it does not. This sieve is not a gesture of the blackboard: periodical cicadas (*Magicicada*) emerge in cycles of 13 or 17 years —primes— because a cycle coprime with that of its predators maximizes the least common multiple and minimizes coincidences (Markus and Goles 2002; Webb 2001; Williams and Simon 1995). No insect knows that 13 is prime, just as no planet knows that 8:5 is coprime. Coprimality is not intention: it is the minimal topological condition for persisting.

The angular atom: the 18. The polar map has twenty rays separated by eighteen degrees. The 18 is the whole step that divides 360 with no remainder; the angles of the family —36°, 72°, 108°— are its integer multiples, and it is not a multiple of any of them. Its algebra is delicate: since 5×18°=90°, the tangent of 18° is a root of the polynomial $5t\ - 10t^2 + 1$, with closed form $\tan(18°) = \sqrt{(25-10\sqrt{5})}/5$. There the 5 is not decorative: wherever tan(18°) appears, the structure of five is already present even if no pentagon is drawn. In the continuum, that tangent demands infinite decimals; in the grammar, the 18 is a whole step that divides 360 with no remainder — a number of infinite decimals on the blackboard that is here exact in integers: the thesis of this work embodied in a number. Literature draws the forms; this work draws the instrument. The 18 will be the protagonist of the operation (§5), without having raised its voice until then.

The ratio of the knot: _ico. The constant ratio of the icosagon is the polar compass: twenty rays sweep the full 360°, and the polar half-turn travels 10 sides of the icosagon over 1 radius. In decimal writing that ratio reads "ten to one"; in the native count, **11:1** — because the symbol following 9 is 11 (10 does not exist). Same ratio, another writing, and neither is a design choice: it is what the 20-ray polar map gives. The decimal 3.12869 appears only if the ratio passes through the customs to decimal ($20 \cdot \sin(\ /20)$ — the verification against the continuum): the icosagon is low resolution, and the decimal is the impression the blackboard produces of a finite ratio. It is not an approximate and does not replace : remains where continuous geometry needs it; inside the engine we do not need it as an irrational. The same map produces : five equidistant knots on the twenty rays always determine the regular pentagon —vertices {0, 4, 8, 12, 16}, separated by 4 rays: 5 × 4 = 20—, and upon it the diagonal/side ratio is , invariant at any radius (`gsf_constant_ratio.py`, appendix). The perfect circle belongs to the continuum: the engine neither materializes nor needs it — it does not accumulate a galaxy of infinite decimals.

Irrationals emerge, they are not injected. Neither , nor , nor √5 come through the door. The grammar produces those relations if the task requires them, and the customs measures them when the question asks for them (§4). The square root is not computed: it is counted (the invariant area = r^2 + debt of §5; r emerges from the count). It is the gesture of this work, in one line: structure is not imported, it is discovered.

Lineage, declared. The 18 is not ours. Euclid builds it implicitly in the decagon; Ptolemy tabulates it explicitly; Archimedes uses it as a step toward (_ico = 3.12869 is the icosagon of Archimedes); Gauss formalizes it algebraically. What this work claims is not the atom: it is using it as a unit of work instead of as an angle of figure.

1.5 4. The machine

Bit-by-bit reversibility. Experiment E8 (the table in §1) shows that the engine is strictly reversible: ten million steps forward and back, zero drift. The 64-bit float accumulates 3.61×10 in the same regime — drift that, in long simulations, degrades the model.

The journey does not forget. The complete journey was repeated 150,015 times, contracting and expanding levels (99→33→11→3→1 and back): zero failures, from 1,005 to 50,005 steps — beyond that range, the claim is extrapolation, not measurement (`e13_reversibility_scaled.py`). Reversibility is structural, not accidental.

The customs in streaming. The only operation that could be tempted to materialize the complete integer is the translation at the output. In the grammar that conversion also processes in streaming, with register per operation less than or equal to 90 (7 bits), without loading the integer — verified up to the grain of rice k25 (10 ^{2}, 22 cells) and the periods of the round (§7) (experiment E17; `e17_streaming_customs_7bits.py`).

The engine does not use sines or cosines. Trigonometry does not act in the engine nor in the customs: the conversion to the world is pure integer arithmetic (Horner in base 90, register per operation 90). Sines and cosines appear only in the verification layer against the continuous expectation and in rendering — never in the flow (E17 verified: zero sin/cos/tan/sqrt in customs and engine). The dependence on the decimal to ”go outside” is a crutch; the FSG removes it.

State integrity. The exactness of the engine confers three data-integrity properties. The independent stress test (`state_integrity_stress.py`, re-stowage 24-Aug-2026) delimited them with window:

- *Reversibility conditioned on the exact initial state.* 93,045 checks (3

scales × 3 rotors), zero failures: the inverse exists and is exact when the complete state is known.

- *Deterministic sensitivity to perturbation (local avalanche).* Altering a

single state remainder breaks between 1 and 4 adjacent positions of the reconstruction (the altered step and its neighbors toward the segment cut), in a **deterministic** way: the same alteration produces the same pattern. The avalanche is local — the segment cut acts as a firewall; alterations in distinct segments do not overlap. Verified at three scales (1,005 to 25,005 steps/rotor).

- *Irreversibility of the lost state.* Without the saved state, the

reconstruction differs in 100% of the positions (the three scales).

The window is honest: **these are preliminary integrity properties, not a security proof.** The avalanche is local (not global as in a cipher), the state space is small, and they are not offered as a replacement for a standard security mechanism — only as the finding that exact integer arithmetic generates these structures without asking for them. Formal analysis is future work (§9).

1.6 5. The snail

The machine leaves the computer and meets a body.

The antecedent that breathes. Before the snail there was the flower: five rings that grow with a breathing rule where debt accumulates the odds and the radius rises when it crosses them. The invariant is exact: area = r^2 + debt. There is no square root, no stored area, no division — only addition, subtraction and one comparison: the root without sqrt, the division without dividing. Twenty-five thousand checks, zero failures (E10). Debt here is the generalized § cell: what is left over is carried until it saturates.

The body that ages. The snail is the flower with a single petal: a ring that grows and turns (window: level 0 of experiment E14 — the clock runs the 90 labels of the icosagon). The step advances three rays —54°, the ray where the sine is exactly /2— and every twenty steps the body returns to ray zero with a larger radius. The five returns of the trace are collinear. **The close of the cycle does not repeat: it ages.** The body is not where it was: it is one cycle older. The spatial reading of time says it in matter: in the shell, what looks newer —the tip, the small part— is the oldest (the apex, the embryonic shell); what looks older —the edge, where the body is— is the newest. The radius is a time axis: the snail carries its entire history, from the ancient tip to the new edge (biology documents this with sclerochronology — §21.3 of the laboratory notebook; we do not claim to explain the biological snail, §11). Replica: 100/100 exact on the native engine (`e14_replicate_trace.py`, appendix).

Why does it not close? Because of a local defect. The 54° step stays one ray —18°— short of the 72° step that would close the pentagonal loop. Fifty-four plus eighteen is seventy-two: the loop closes; it does not close because that single 18 is missing. The classical reading speaks of a defect of two rays against the drawn figure; the engine shows a defect against itself, locally: the snail does not care to close against an external figure (that is a contract with the plane, the tile); it cares about itself. That 18 —the one the figure reading does not see— is the one that does not close with itself, and the one nobody looks at (§3).

The consequence is the form: **the spiral is the mouth that the closed figure does not have.** The defect is the printer's engine: without defect there is no growth; a hexagon closes and does not spiral. Spatial debt accumulates, saturates the cell, and the radius makes a forced jump outward — a new layer. The area grows by three per step, the rotation is constant, the squared radius is proportional to the angle: it is a Fermat spiral, not an Archimedean one. The snail ages by count, additively; the real one by proportion. It is, in the vocabulary of this work, an organism of additive arithmetic produced by the engine: the word "organism" describes the dynamics of the system — it does not claim biological kinship; the relation to biology is declared below, with its yardstick.

The record the framework describes. The empirical record of shells documents five phenomena;

the topology of this engine provides the framework free of decimal artifacts to describe them: the radius as chronological axis (sclerochronology); discontinuous growth by saturation of mantle tension (the calcium layer deposits at once); the geometric defect as the engine of the open spiral; emerging from physical optimization, not programmed; and the global form emerging from local rules. It is not analogy — it is operator isomorphism with its explicit map: *biology minimizes local energy; the engine minimizes local debt; both produce discontinuous growth in jumps.* The framework does not create the shell: it describes it without the noise of the decimal, and that is all this work claims. The yardstick is not overplayed: the yardstick for explaining a biological process is predicting its transitions, and we have not reached it yet. Here we only show that an integer-arithmetic engine describes, with a single operator, what biology does with millions of mantle cells.

1.7 6. The budget

The jewel is a theorem.

The problem. Five rings rotate on the twenty rays, each with its rational cadence (denominator N). When do they all align again at once? Intuition would say: when the clocks coincided — and one computes position by position, decimal by decimal.

The theorem. For k rings with denominators bounded by N, the simultaneous return never exceeds **T 20 · M(N)**, with M(N) = p^ log_p N (product of the maximal prime powers up to N). Twenty times the largest achievable common multiple: twenty, because the angular return is measured in turns of the icosagon. It is an exact bound, not a heuristic. The optimal configuration (N=17, k=5, denominators {16, 15, 17, 13, 11}) gives a period of **11,668,800 beats**, with a strong prediction: exact return at that beat, not one before, not one after.

The falsification. The theorem was attacked against itself in four layers: lemmas, theorems, optima by exhaustive enumeration and the strong prediction. **17/17 rows correct: the theory was not falsified.** The theorem's clock is reversible (6/6) and the validation without running confirms the formula (5/5).

The optimum is not intuition. Five slots, denominators up to 17: which numbers should be kept? Common sense proposes the largest available — {9, 7, 11, 13, 17} (9 is not prime, but it is 3^2 and preserves factor independence). That recipe produces a return of 3,063,060 beats. The theorem corrects it: with scarce slots, factor diversity defeats the depth of a single large factor. The optimal sequence is {16, 15, 17, 13, 11}: 16 (2) is the maximal power of 2 within the cap; 15 packs two factors into a single slot. The notebook sums it up: *15 = 3 · 5 travels free where 9 does not* — two factors in the slot of one. The return jumps to 11,668,800 — almost four times the recipe of large powers.

Power grows without enlarging the numbers: a single factor raised up to 243 gives 4,860 beats; the diversity of small primes {3, 7, 11, 13, 17} up to 17 gives 1,021,020 — two hundred times more, with numbers fourteen times smaller (notebook §23).

Origin of the theorem. The theorem was brought by an artificial co-investigator (the node Z of the notebook) with its strong prediction (2026-08-19; complete lineage preserved); and the same theorem falsified the previous recipe its own author brought ("distinct powers"): the confession is in the notebook. The theory was never falsified (17/17, above); what fell was a recipe.

The grain of rice. The N=99 case takes the bound to its largest consequence. With five one-byte cells —less space than a grain of rice in memory— and denominators {97, 89, 83, 79, 73}, the simultaneous return is **82,645,608,260 exact beats**, verified by pure arithmetic without simulating (closed formula, sampling of 10,101 checks with zero discrepancies, generic identity in 50,000 tuples). Extended to k=10, the bound exceeds the seconds since the Big Bang (~7.5×10^{1}); at k=25, ~1.4×10^{2}. **The numbers that cannot be represented are born of minimal combinations of small integers.** Incommensurability is not achieved by inventing digits: it is achieved by bounding the denominator and letting arithmetic work. Arithmetic has no corner where it changes.

Disproportion emerges from sobriety.

1.8 7. The round

The budget is a theorem of rings; the year is one more ring.

A calendar as input. The system admits periodic calendars, and the map enters the theorem as another gear. With the 360-day year as an additional ring, the period of the polar turn triples: the least common multiple of {320, 300, 340, 260, 220, 360} is **35,006,400 = 3 × 11,668,800**. The year holds three lives of the snail of §5, and all three fall on calendared days: the return of the five rings falls every 11,668,800 beats, which modulo 360 is day 120; the lives of the snail fall on days 120, 240 and 360 — three lives of the snail, one complete conjunction. The calendars behave as § cells of the icosagon: the 260-day ritual cycle is 13×20 (no residue); the 365-day solar year is 18×20 **with residue 5** — the five surplus days, the Wayeb', are a calendared residue: they are not discarded, they are calendared; the 584-day Venus cycle is 29×20 with residue 4. The same honesty with what is left over, on another substrate.

The weld is a prime. Seventy-three is the gear that welds the cycles: 365 = 5×73 and 584 = 8×73, so that 5×584 = 8×365 — exact. Five years of Venus are eight years of Earth, in pure 8:5 proportion, without decimals. The least common multiple of {260, 365, 584} is 37,960, and the complete return of the three cycles falls at **T = 759,200 beats = 104 Haab' = 146 Tzolk'in = 65 Venus** — the round. While it unfolds, the encounters of Venus and Earth draw in the sky the pentagram on the rays {0, 8, 16, 4, 12} of the same polar map: the canonical pentagon written in the sky.

Sovereignty of the record. The Maya recorded these cycles; we do not claim they handled the concepts with which we read them. Coprime, quasiperiodicity, § cells: that language is ours. What was theirs was accumulated observation and counting — a sophistication the record itself lets be seen. That our formalism replicates what they noted is not the revelation of a secret code: it is the verification, with another tool, of that precision. A civilization without a supercomputer calculated the cycle by hand over generations; the same result fits in five one-byte cells (§6). The awe is not about technology: it is about arithmetic.

The orbital context, as a cited standard reading (not our measurement). Venus and Earth did not sign a pact. Celestial mechanics documents that the frequencies that persist are those that avoid resonances of small numbers: if two cycles resonated, accumulation would perturb the orbits until they fell apart; the configurations that survive are those that do not coincide often (Poincaré 1890; Kirkwood 1866; KAM theorem: Kolmogorov 1954, Arnold 1963, Moser 1962). In the asteroid belt, the Kirkwood gaps sit exactly at the resonances with Jupiter. That is the same filter coprimality describes in integer arithmetic (§3) and the same one biology shows in the cicadas. The round does not validate the FSG; the FSG shows that the round was a case of the budget.

Archaeoastronomy measured that the platforms and windows of El Caracol, in Chichén Itzá, are deliberately oriented to horizon events of the Sun and of Venus (Aveni, Gibbs and Hartung 1975; Aveni 2001). The pentagram is not a house contrivance: it invokes the body that Mesoamerican astronomy observed first — the evidence of its observation is measured and published. And in biology the same sieve is seen: periodical cicadas survive because their prime cycles do not resonate with those of their predators (§3).

1.9 8. Operational impact

The theory is used. Three immediate consequences, in technical register:

Minimal configuration. Each cell fits in seven bits; an eight-bit microcontroller processes numbers of arbitrary length in streaming, without loading the complete integer (E17, §4). For minimal-resource hardware —sensors, edge devices, embedded logic— exact arithmetic and limited precision arithmetic cease to be mutually exclusive: exactness fits where previously only approximation fit.

Local avalanche. The deterministic sensitivity to perturbation (§4) has a direct reading for data integrity: altering a single value of the state breaks —deterministically and locally— the adjacent positions of the reconstruction. It is the signature of a computation that does not sweep differences: either it reconstructs with the exact state, or the reconstruction differs immediately and visibly. It is not a formal security mechanism; it is the finding that exactness does not hide its discrepancies.

The declared collapse. When the FSG emits to the world (§3), it collapses once, at the end, with a known bound: at the pixel render, the worst observed deviation is 0.4758 px on a construction ceiling of 0.5 px (half a cell), also verified on 600-px and 2400-px canvases — the bound does not depend on the resolution of the support (experiment E20; `e20_octant_catastro.py`; stamp computed by the code itself, never edited by hand). Conventional rendering collapses at every chained operation without a record; the FSG collapses once and declares it.

1.10 9. Horizons and conjectures

Conjecture: the residue as emergent dimensionality. The § cell accumulates, in each operation, exactly what the continuous mathematical expression describes as a lower dimension. Whether that correspondence is an artifact of notation or a property of integer arithmetic is an open question. **Written falsification criterion:** the conjecture falls if an integer operation is found whose § cell accumulates a value with no correlate in any formulation of the same problem — or if the residue limit, as depth grows, stops following the cell bound. Until it is falsified or proven, it is called a conjecture.

Future work. Native metrics that measure the engine in its own language, not only in its decimal projection. Formal analysis of the integrity properties (§4). Quasicrystals, complete calendars, and the graphic customs as a standard. As for what this collective sees with the general use of language models and does not develop here: it is matter for coming deliveries — this document is the first.

1.11 10. Methods

This work is the first delivery of a hybrid collective: N-version verification among independent implementations of human nodes and language models; whoever generates does not audit; each node records its log separately (mirror logs; complete protocol in the collective's methodological appendix, with the legal-responsibility statement and the collaboration method).

The code pieces rest with their hash and are run to reproduce each table (Appendix A; the epigraph of this version comes from the internal tradition of the collective). Every assertion in this document carries a declared window: exact to the resolution the task declares, valid in the measured regime. References to external verifications are marked as such.

1.12 11. Statement of boundaries

What this work does not claim:

- It does not claim to have discovered the 18°, nor tan(18°), nor , nor

aperiodic tilings, nor phyllotaxis. Literature already knows them; this work shows where they operate without being injected.

- It does not claim the continuum is false: it holds that it is unnecessary

for what we measure. It does not claim infinity does not exist: a value without a bound is not a result, it is a warning that the model left its range.

- It does not explain any biological process: the yardstick for explaining is

predicting transitions, and we have not reached it yet (§5).

- It does not attribute to the Maya our mathematical vocabulary (§7).
- It does not claim an orbital pact: it cites the standard reading of

celestial mechanics (Poincaré, Kirkwood, KAM) as context, not as its own measurement.

- It does not deny a priori that numerical convergences are coincidences:

even so, they are useful tools.

1.13 12. Appendices

1.13.1 Appendix A — Script index (claim → script → output)

Code license: Apache License 2.0 — not covered by the CC BY 4.0 of the text.

Each stowed and verified piece (code and output hash in the work repository): `e8_reversibility.py` (drift 0 vs 3.61e-9), `e13_reversibility_scaled.py` (150,015 checks, 0 failures; sensitivity; irreversibility), `e15_falsation.py` (17/17), `e15_reversibility.py` (6/6), `e15_validation_without_running.py` (grain of rice), `e16_periodic_calendar.py` (35,006,400), `e16_ronda.py` (759,200), `e14_replicate_trace.py` (snail 100/100), `e10_respiration_native_motor.py` (25,000 checks), `e17_streaming_customs_7bits.py` (7-bit customs), `state_integrity_stress.py` (avalanche 1–4, irreversibility 100%), `e20_octant_catastro.py` (Δr 0.4758 px), plus the grammar modules (`modelo99.py`, `gsf_customs.py`, `gsf_contraction.py`, `gsf_galaxy.py`, `gsf_constant_ratio.py`, `gsf_beacons.py`, `gsf_labeler.py`, `gsf_division.py`, `gsf_pixel99_topology.py`).

1.13.2 Appendix B — Brief glossary of notation

- **Tuple:** the number as a sequence of cells (depth, not width).
- **Cap:** the maximum a cell admits before saturating (instance: 90

positions, label 99).

- **§ cell:** surplus cell; keeps the exact residue of a division.
- **Debt:** in the radial system, the residue carried until it saturates

(the `deuda` identifier in the scripts).

- **Customs:** the translation frontier to decimal, where the question

declares its resolution.

- **Bounded register:** the working state per operation, always 90 (7

bits).

- **Beat:** one step of the system (an addition, a subtraction, a

comparison).

1.13.3 Appendix C — References

- Aveni, A. F., Gibbs, S. L., & Hartung, H. (1975). The Caracol Tower at

Chichen Itza: An Ancient Astronomical Observatory? *Science* 188(4192), 977–985.

- Aveni, A. F. (2001). *Skywatchers.* University of Texas Press.
- Bricker, V. R., & Bricker, H. M. (2011). *Astronomy in the Maya Codices.*

Brill.

- Haller, G. (2015). Lagrangian coherent structures. *Annual Review of Fluid Mechanics* 47, 137–162.
- Kolmogorov, A. N. (1954). On the preservation of conditionally periodic motions. *Dokl. Akad. Nauk SSSR* 98, 527–530.
- Arnold, V. I. (1963). Small denominators. *Russian Mathematical Surveys* 18(6), 9–36.
- Moser, J. (1962). On invariant curves of area-preserving mappings. *Nachr. Akad. Wiss. Göttingen* IIa, 1–20.
- Markus, M., & Goles, E. (2002). Cicadas showing up after a prime number of years. *The Mathematical Intelligencer* 24, 30–32.
- Webb, G. F. (2001). The prime number periodical cicada problem. *Discrete and Continuous Dynamical Systems* 1(3), 387–399.
- Williams, K. S., & Simon, C. (1995). The ecology, behavior, and evolution of periodical cicadas. *Annual Review of Entomology* 40, 269–295.
- Palmer, T. N. (2001). A nonlinear dynamical perspective on model error. *Quarterly Journal of the Royal Meteorological Society* 127(572), 279–304. DOI 10.1002/qj.49712757202.
- Poincaré, H. (1890). Sur le problème des trois corps et les équations de la dynamique. *Acta Mathematica* 13, 1–270.
- Kirkwood, D. (1866). On the theory of meteors. *American Journal of Science* (under editorial verification).
- Förstemann, E. (1880). Commentary on the Dresden Codex (under editorial verification).

What is lost, exactly, when reversibility is gained. The cell does not answer: it only saturates, and on saturating it rises a level. The code runs; whoever it serves, let them use it.